

# POWER CHARACTERISATION OF A MESFET AMPLIFIER USING SMALL SIGNAL MEASUREMENTS AND VOLTERRA SERIES

G. Lambrianou, C.S. Aitchison\*,

Department of Electronics, Chelsea College, (University of London)  
Pulton Place, London SW6 5PR, U.K.

\*now at ERA Technology Ltd., Leatherhead, Surrey, KT22 7SA, U.K.

## Abstract

The power performance of a microwave GaAs MESFET amplifier is predicted from small signal measurements based on the Volterra Series representation. Expressions for the non-linear power gain frequency response, the output power and gain compression factor are presented. The major sources of gain compression are identified. Experimental verification of the theory is presented by means of a 1.3 $\mu$ m gate MESFET amplifier at 7.8GHz.

## Introduction

The Volterra Series representation is well established as one of the most powerful techniques for analysing non-linear circuits with mild non-linearities[1]. Even though considerable work has been done in the area of MESFET power amplifiers using this series, most of the previously published work only dealt with the problem of third-order intermodulation distortion up to S-band frequencies [2],[3], without further predicting the full power performance of the amplifier at higher frequencies. Other methods like the one by Tucker [4] involved large signal measurements.

This paper investigates the full performance of a single stage MESFET amplifier at X-band based on small signal measurements only, using a non-linear bilateral circuit model. A third-order analysis together with its limitations is presented based on the Volterra Series representation. Analytical expressions are obtained which relate the non-linear power gain frequency response, the gain compression and the non-linear output power with the load impedance and the input power level. The analytical results are compared with measurements on an experimental GaAs MESFET amplifier at X-band. Finally the effect of device non-linearities on the power performance of the amplifier are identified individually.

## MESFET Amplifier Non-Linear Model

The non-linear circuit model of the conjugately matched amplifier employing the MESFET device,

which we have used, is shown in figure 1(a). The device model contains three non-linearities, the gate capacitance,  $C_{gs}$ , the transconductance,  $g_m$ , and the output conductance,  $g_o$ . The non-linearities have physical origins and are based both on physical model and measurements. The physical model is derived primarily from the JFET model of [5],[6] with the necessary modifications to determine the MESFET non-linearities bias dependence ( $g_m$ - $V_{GS}$  for constant  $V_{DS}$ ,  $g_o$ - $V_{DS}$  for constant  $V_{GS}$  and  $C_{gs}$ - $V_{GS}$ ). Small signal measurements are carried out to confirm the theory referred to above. The choice of the Volterra series technique requires that the non-linear elements must be presented by non-linear voltage-controlled current generators using the first three terms of the Taylor series expansion of the characteristics above the operating point, as explained in [3]. The non-linear current sources are superimposed on the linear circuit in the form of additional current generators. (Fig. 1(a)).

## Non-Linear Analysis of the MESFET Amplifier

The summary of the non-linear analysis is given as follows. Consider the non-linear amplifier given in Fig. 1(a) with source and load impedance given as

$$Z_{ms} = R_s + j X_s \quad (1)$$

$$Z_{mL} = R_L + j X_L \quad (2)$$

When the input signal is of the form  $V_i(t) = v_i \cos \omega_1 t$ , the circuit non-linear transfer function at  $\omega_1$ ,  $T_{NL}(\omega_1)$ , is of the form

$$T_{NL}(\omega_1) = H_{01}(\omega_1) + 3 P_{av}(\omega_1) \operatorname{Re} [Z_{ms}] H_{03}(\omega_1, \omega_1, -\omega_1) + \dots \quad (3)$$

where  $P_{av}(\omega_1) = \frac{|\bar{v}_i|^2}{4 \operatorname{Re} [Z_{ms}]}$  is the available input power,

$H_{01}(\omega_1)$  and  $H_{03}(\omega_1, \omega_1, -\omega_1)$  are the Volterra non-linear transfer functions and can be calculated using the technique described in [1] and [7]. Also the non-linear output power at  $\omega_1$ ,  $P_{ONL}(\omega_1)$  can be expressed as

$$P_{ONL}(\omega_1) = \frac{|\bar{v}_{ONL}(\omega_1)|^2}{\operatorname{Re} [Z_{mL}]} \quad (4)$$

where  $V_{ONL}(\omega_1)$  is the output voltage across the real part of the matched load  $Z_{mL}$ . Using equations (3) and (4) the amplifier power non-linear gain at  $\omega_1$ ,  $G_{PNL}(\omega_1)$  in decibels is

$$G_{PNL}(\omega_1) = 10 \log_{10} |T_{NL}(\omega_1)|^2 + 10 \log_{10} \left| \frac{R_L}{R_L + jX_L(\omega_1)} \right|^2 + 10 \log_{10} \left| 4 \operatorname{Re} \left\{ \frac{Z_{ms}}{Z_{mL}} \right\} \right| \quad (5)$$

It is seen from (5) that the non-linear output power at  $\omega_1$  is of the form

$$P_{ONL}(\omega_1) = G_{PNL}(\omega_1) \cdot P_{av}(\omega_1) \quad (6)$$

By definition the Voltage compression ratio,  $K(\omega_1)$  is the deviation of amplifier voltage gain from its small signal value [4]. This is derived from (3) and (4) and is found to be:

$$|K(\omega_1)| \simeq 1 + 3 P_{av}(\omega_1) \operatorname{Re}[Z_{ms}] \operatorname{Re} \left\{ \frac{H_{03}(\omega_1, \omega_1, -\omega_1)}{H_{01}(\omega_1)} \right\} \quad (7)$$

In terms of power gain the corresponding parameter is  $|K(\omega_1)|^2$  and is called the gain compression factor. Equations (5) to (7) express the non-linear power gain, the output power and the voltage compression as a function of the amplifier non-linear transfer functions, the input frequency, and the terminated loads, for any arbitrary input power level.

The theory presented is subject to a limitation which is a function of the input power and device parameters, and is given in the form of the following inequality.

$$\frac{\operatorname{Re} \left\{ \frac{H_{01}(\omega_1)}{H_{03}(\omega_1, \omega_1, -\omega_1)} \right\}}{3 \operatorname{Re}[Z_{ms}]} > P_{av}(\omega_1) > 0 \quad (8)$$

This last equation gives the range of the input power where the theory is valid.

#### Experimental Verification

The first experiment is to compare the Volterra analysis in the frequency domain with the time domain analysis using the computer program SPICE 2. Fig. 2 shows the comparison between calculated (time and frequency domain) and measured single tone output power versus input power for 7.8GHz amplifier terminated with a 50Ω load and source impedance. The next experimental test is to fully characterise the power performance of a conjugately matched amplifier built on an alumina substrate using  $\lambda/4$  matching sections as shown in the photograph of Fig. 1(b). Fig. 3 shows the measured and calculated non-linear power gain as a function of the input frequency for different input power levels. The calculated and measured results of the output power gain as a function of input power is presented in Fig. 4. The gain compression curves, as a function of the input power, for both the conjugate match and 50Ω load amplifiers, is shown in Fig. 5. The figures from (2) to

(5) serve to demonstrate the ability of the model in conjunction with the theory to predict the power performance of a non-linear amplifier from small signal measurements. Measured and predicted points using the non-linear model are substantially coincident, although a small gain discrepancy with a maximum of 0.5dB at the highest input power level is observed. These are due to contributions from higher order transfer functions which are not included in this analysis.

The relative contribution made by each non-linearity to the total power performance can be assessed by including the non-linear elements one at a time in the non-linear model and computing the input-output power characteristics. The results for the conjugated match amplifier are presented in Fig. 6. The conductance,  $g_o$ , is the dominant non-linearity, which compresses the output power from its linear value by 3.7dB, next is the transconductance,  $g_m$ , with a smaller effect by 0.93dB and finally the  $C_{gs}$  capacitance with almost negligible effect by 0.35dB with 10dBm input power. This behaviour suggests that for the design of an amplifier for optimum output power the drain conductance will be the dominant factor for attention.

#### Conclusion

The power performance of a microwave MESFET amplifier at arbitrary input power levels, from small-signal measurements, has been successfully predicted, using the Volterra series. Simple expressions are obtained for the non-linear power gain, the output power, the compression gain, together with their corresponding limitations. The model enables us to identify the major causes of the gain compression for a device embedded in an amplifier. This analysis may readily be extended to obtain the load value for optimum output power and may find applications in high power amplifier and oscillator design.

#### Acknowledgement

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#### References

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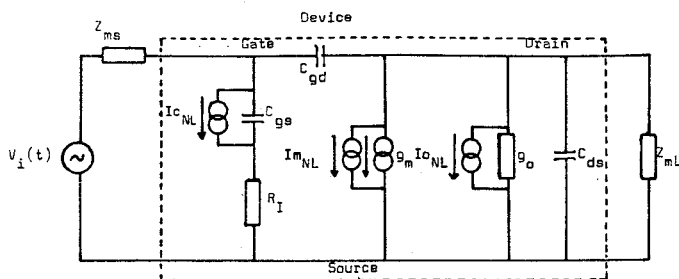


Fig. 1(a) Non-linear circuit representation of the MESFET amplifier under large signal conditions

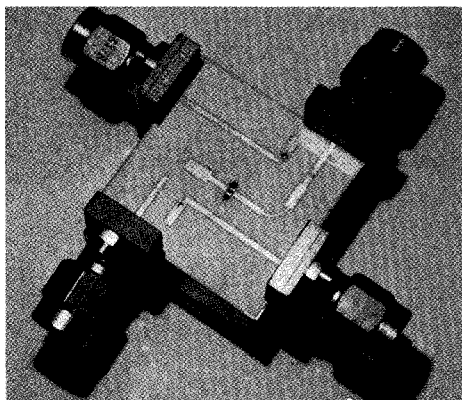


Fig. 1(b) Internal view of one stage conjugately matched amplifier at 7.8GHz

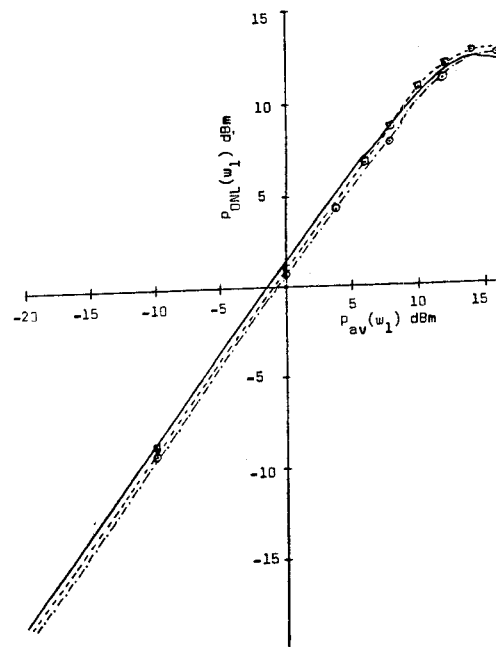


Fig. 2 Comparison between calculated and measured single tone output power versus input power for the device terminated with 50 ohms load impedance at 7.8GHz

Calculated from computer ---○---  
Calculated from the Volterra series ———  
Measured ---□---

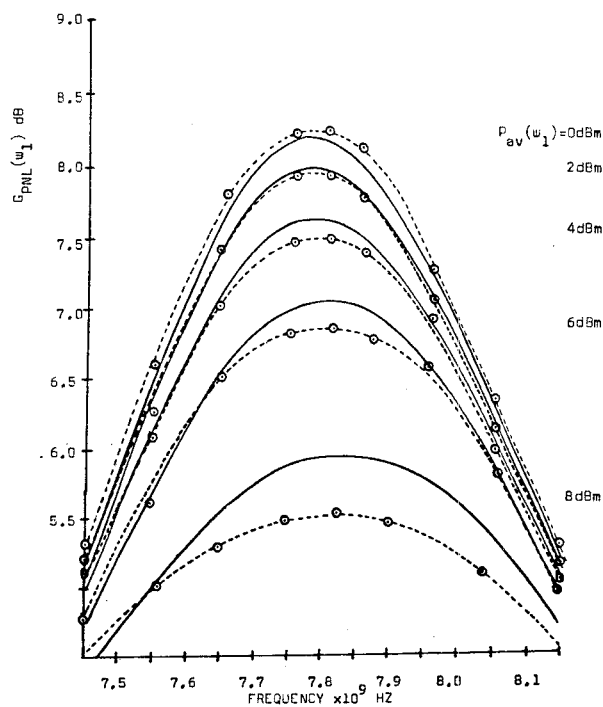


Fig. 3 Comparison between calculated and measured single tone power gain frequency response for a conjugated match amplifier for different input power level at 7.8GHz

Calculated using the Volterra series ———  
Measured ---○---

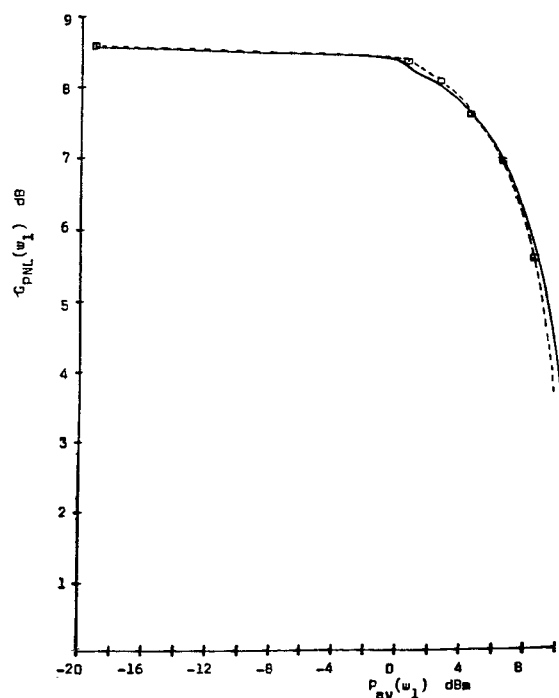


Fig. 4 Calculated and measured single tone output power gain versus input power at 7.8GHz  
Calculated using the Volterra series —  
Measured - -□- -

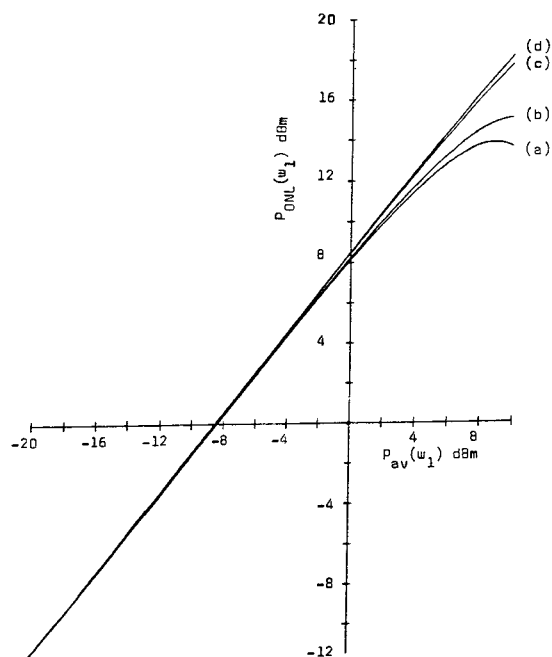


Fig. 6 Calculated single tone output power versus input power for the conjugated match amplifier at 7.8GHz  
(a) Including all non-linearities  
(b) Including  $g_o$  non-linearity only  
(c) Including  $g_m$  non-linearity only  
(d) Including  $c_{gs}$  non-linearity only.

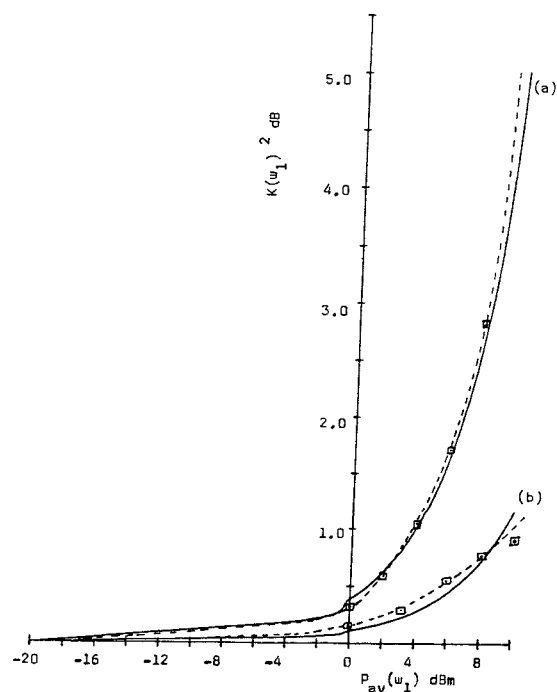


Fig. 5 Calculated and measured gain compression at 7.8 GHz  
(a) For the conjugated match amplifier  
(b) For the device terminated with 50ohms load  
Calculated —  
Measured - -□- -